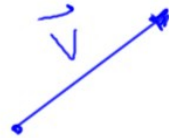




0. Conceptos Previos

1) Vectores:



Dirección

Sentido u Orientación

Tamaño ó Módulo

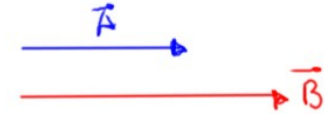
$$|\vec{v}| = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

3) Operaciones de Vectores:

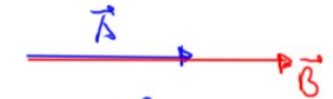
- 1) Suma de Vectores
- 2) Resta de Vectores
- 3) Multiplicación por escalar

2) Tipos de Vectores

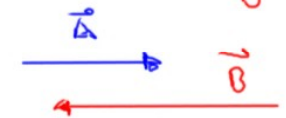
Paralelos:



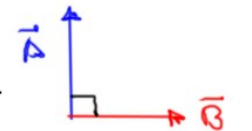
Colineales:



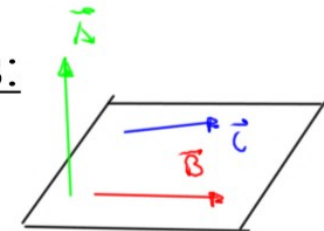
Antiparalelos:



Perpendiculares:



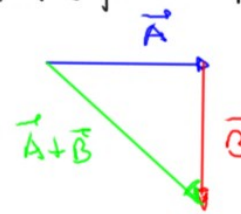
Coplanares:



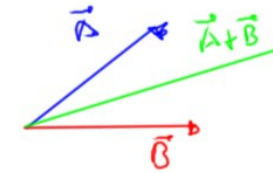


1) Suma de Vectores: $\vec{A} = (a\hat{x} + b\hat{y} + c\hat{z})$, $\vec{B} = (d\hat{x} + e\hat{y} + f\hat{z})$

$$\vec{A} + \vec{B} = ((a+d)\hat{x} + (b+e)\hat{y} + (c+f)\hat{z})$$

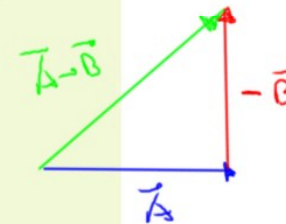


Ley del paralelogramo



2) Resta de Vectores:

$$\vec{A} + (-\vec{B}) = ((a-d)\hat{x} + (b-e)\hat{y} + (c-f)\hat{z})$$



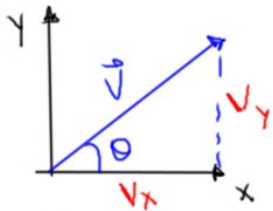
3) Multiplicación por escalar: $\alpha\vec{A} = \alpha(a\hat{x} + b\hat{y} + c\hat{z})$





4) Vectores en 2D:

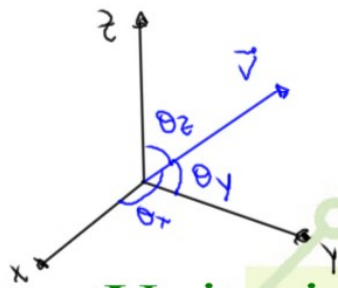
$$\begin{aligned} \hat{x} &= \hat{i} \\ \hat{y} &= \hat{j} \\ \hat{z} &= \hat{k} \end{aligned}$$



$$\begin{aligned} \vec{V}_x &= |\vec{V}| \cos \theta \hat{i} \\ \vec{V}_y &= |\vec{V}| \sin \theta \hat{j} \end{aligned}$$

$$\theta = \tan^{-1} \left(\frac{V_y}{V_x} \right)$$

5) Vectores en 3D:



$$\begin{aligned} \vec{V}_x &= |\vec{V}| \cos \theta_x \hat{i} \\ \vec{V}_y &= |\vec{V}| \cos \theta_y \hat{j} \\ \vec{V}_z &= |\vec{V}| \cos \theta_z \hat{k} \end{aligned}$$

$$\theta_x = \cos^{-1} \left(\frac{V_x}{|\vec{V}|} \right)$$

$$\cos^2 \theta_x + \cos^2 \theta_y + \cos^2 \theta_z = 1$$

(Cosenos directores)

6) Vector Unitario: $|\hat{V}| = 1$

$$\hat{V} = \frac{\vec{V}}{|\vec{V}|}$$





7) Producto escalar: $\vec{A} \perp \vec{B} \Rightarrow \vec{A} \cdot \vec{B} = 0$

$$\theta = \cos^{-1} \left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} \right) \quad (\text{Ángulo entre dos vectores})$$

Definición: $\vec{A} \cdot \vec{B} = |\vec{A}| |\vec{B}| \cos \theta$

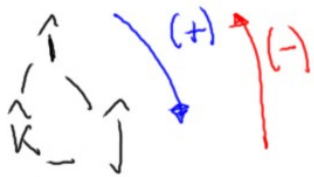
Forma Analítica: $\vec{A} \cdot \vec{B} = (a_x b_x) + (a_y b_y) + (a_z b_z) = \text{Número (Escalar)}$

8) Producto Vectorial: $\vec{A} \parallel \vec{B} \Rightarrow \vec{A} \times \vec{B} = 0$ Regla de la mano derecha:

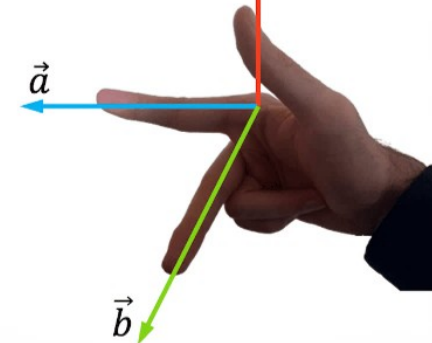
Definición: $\vec{A} \times \vec{B} = |\vec{A}| |\vec{B}| \sin \theta$

Forma Analítica:

$$\vec{A} \times \vec{B} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix} = \hat{i} [(a_y b_z) - (a_z b_y)] - \hat{j} [(a_x b_z) - (a_z b_x)] + \hat{k} [(a_x b_y) - (a_y b_x)]$$



$\vec{a} \times \vec{b} = \odot$ Positivo
 $\vec{b} \times \vec{a} = \otimes$ Negativo





9) Movimiento Circular:

Velocidad Tangencial: $V = \omega r$ [m/s]

Posición Angular: $\Theta = \int \omega dt$ [rad]

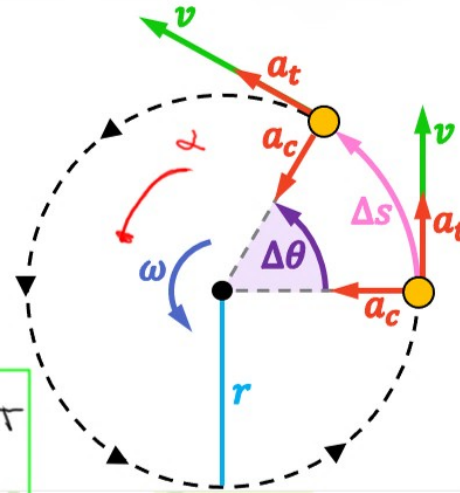
Velocidad Angular: $\omega = \frac{d\theta}{dt}$, $\omega = \int \alpha dt$
 $\omega \rightarrow$ [rad/s]

Aceleración Angular: $\alpha = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2}$ [rad/s²]

Aceleración Centrípetra: $a_c = \frac{v^2}{r} = \omega^2 r$ [m/s²]

Aceleración Tangencial:

$$a_T = \alpha r \quad [m/s^2]$$



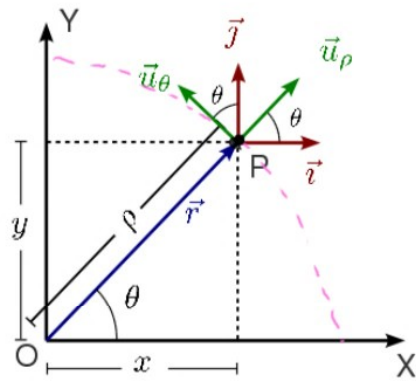
Arco de una circunferencia:

$$\Delta s = r \Delta \theta$$





10) Coordenadas Polares: $(\hat{\rho}, \hat{\theta}) \Rightarrow (\hat{r}, \hat{\theta})$



$$\hat{\rho} = \cos\theta \hat{x} + \sin\theta \hat{y}$$
$$\hat{\theta} = -\sin\theta \hat{x} + \cos\theta \hat{y}$$

